

Name: _____

University of Michigan Physics Department Graduate Qualifying Examination

Part I - Classical Physics

Saturday, January 5, 2008 9:00am-1:00pm

This is a closed book exam - but you may use the materials provided at the exam. If you need to make an assumption or estimate, indicate it clearly. Show your work in an organized manner to receive partial credit for it.

You must answer the first 8 obligatory questions and two of the optional four questions. Indicate which of the latter you wish us to grade (e.g., circle the question number). We will only grade the indicated optional questions. Good Luck.

SOME FUNDAMENTAL CONSTANTS IN CONVENIENT UNITS

speed of light	$c = 2.998 \times 10^8$ m/s
electron charge	$e = 1.602 \times 10^{-19}$ C
Planck's constant	$h = 6.626 \times 10^{-34}$ J · s = 4.136×10^{-15} eV · s
	$\hbar = h/2\pi = 1.055 \times 10^{-34}$ J · s = 0.658×10^{-15} eV · s
Rydberg constant	$R_\infty = 1.097 \times 10^6$ m ⁻¹
Coulomb constant	$k = (4\pi\epsilon_0)^{-1} = 8.988 \times 10^9$ N · m ² /C ²
Universal gas constant	$R = 8.3$ J/K · mol
Avogadro's number	$N_A = 6 \times 10^{23}$ mol ⁻¹
Boltzmann's constant	$k_B = R/N_A = 1.38 \times 10^{-23}$ J/K = 8.617×10^{-5} eV/K
Stefan – Boltzmann constant	$\sigma = 5.6703 \times 10^{-8}$ W/m ² K ⁴
radius of the sun	$R_{sun} = 6.96 \times 10^8$ m
radius of the moon	$R_{moon} = 1.74 \times 10^6$ m
radius of the earth	$R_{earth} = 6.37 \times 10^6$ m
	$G_N = 6.67 \times 10^{-11}$ m ³ /kg/s ² = 6.71×10^{-39} GeV ⁻²

SOME USEFUL CONVERSIONS AND COMBINATIONS

fine structure constant	$\alpha = ke^2/\hbar c = 1/137$
Bohr magneton	$e\hbar/2m_e = 9.27 \times 10^{-24}$ J/T = 5.79×10^{-5} eV/T
	$hc = 19.865 \times 10^{-26}$ J · m = 12.41×10^3 eV · Å = 1241 MeV · fm
	$\hbar c = 3.165 \times 10^{-26}$ J · m = 1973 eV · Å = 197.3 MeV · fm
	$ke^2 = 1.44$ MeV · fm
	$1\text{Å} = 10^{-10}$ m = 10^5 fm
	$1\text{eV} = 1.602 \times 10^{-19}$ J

SOME USEFUL RELATIONS

$$\begin{aligned} \frac{C_V}{Nk} &= 9 (T/\theta_D)^3 \int_0^{\theta_D/T} \frac{x^4 e^x dx}{(e^x - 1)^2} \quad (\text{Debye formula}) \\ &= 3 \left[1 - \frac{1}{20} \left(\frac{\theta_D}{T} \right)^2 + \dots \right] = \frac{12\pi^4}{5} \left(\frac{T}{\theta_D} \right)^3 (1 + \dots) \end{aligned}$$

$$\frac{U}{N} = \frac{3}{5} E_F \left[1 + \frac{5\pi^2}{12} \left(\frac{kT}{E_F} \right)^2 - \frac{\pi^4}{16} \left(\frac{kT}{E_F} \right)^4 + \dots \right] \quad (\text{degenerate electron gas})$$

$$E_F = \frac{\hbar^2}{2m} \left(\frac{3\pi^2 N}{V} \right)^{2/3} \quad (\text{nonrelativistic Fermi energy})$$

$$r = r_o A^{1/3}, \quad r_o = 1.2 \times 10^{-15} m \quad (\text{approximate average nuclear radius})$$

$$\frac{n}{s} + \frac{n'}{s'} = \frac{n' - n}{R} \quad (\text{refraction of paraxial rays})$$

MASSES OF SOME ELEMENTARY PARTICLES

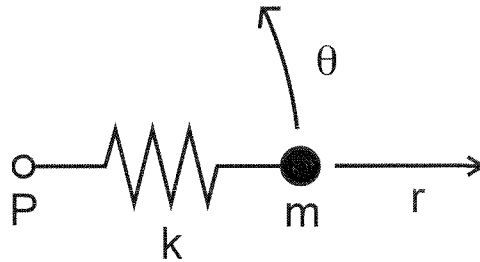
	Rest Mass, m_0 (kg)	$m_0 c^2$ (MeV)
Electron	9.109×10^{-31}	0.511
Proton	1.673×10^{-27}	938.3
Neutron	1.675×10^{-27}	939.6
Atomic mass unit (1 amu)	1.661×10^{-27}	931.5

VISIBLE LIGHT SPECTRUM

300	400	500		600		700(Nanometer)
← Ultraviolet	Violet	Blue	Green	Yellow	Orange	Red Infrared →

PART A: Obligatory Problems

- (Mechanics) On top of a frictionless table we have a massless spring with constant k fixed at point P to the table. At the other end of the spring, a mass m is attached.
 - The spring is stretched slightly and then released. What is the frequency ω of small oscillations?
 - The spring may rotate freely about point P. The mass is pushed in the θ direction, thereby giving it an angular momentum L_z . By what amount is the spring stretched? (Write your answer in terms of L_z and the radial position of the rotating mass.)
 - The revolving mass is then given a small tap in the radial direction. What is the frequency ω of small oscillations?



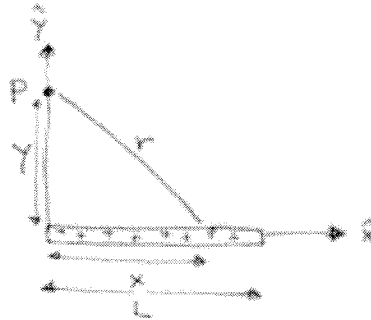
2. (Mechanics) Consider a physical system with a Lagrangian of the form

$$\mathcal{L} = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) - \frac{1}{2}A(x^2 + y^2) + \frac{B}{(x^2 + y^2)^{1/2}} + \frac{C}{2(x^2 + y^2)},$$

where x and y are the coordinates and (A, B, C) are constants. Find the equations of motion and show that angular momentum is conserved.

3. (Mechanics) A plumb-bob, a weight with a pointed tip on the bottom, is hung from a string tied to the top of the leaning tower of Pisa. At equilibrium, does the string point toward the center of earth? If not, where does it point? Assume the earth is a sphere of radius R and the latitude (angle relative to the equator) of Pisa is θ .

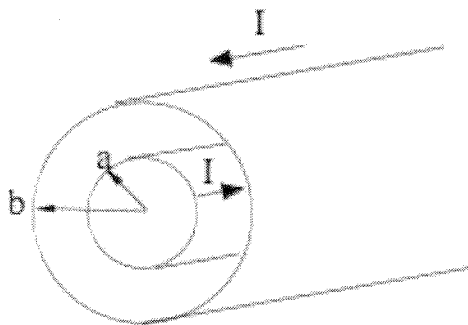
4. (E&M) Consider a non-uniform linear charge density $\rho(x) = ax$, where a is a positive constant, distributed along the x -axis from $x = 0$ to $x = L$.



- (a) Determine the electric potential V at point P .
(b) Discuss the situation for $y \gg L$

5. (E&M) A charge q , is held at a distance d above a grounded infinite conducting plane.
- (a) Calculate the electric field in cartesian coordinates (take the origin to be the point on the plane closest to the charge).
 - (b) What is the force between the charge and the conducting plane? How much energy is required to take a charge from infinitely far away to its current location?
 - (c) Describe what would happen if we release the charge.

6. (E&M) Consider a coaxial cable made using two thin walled conducting tubes of radius a and b . In a typical operation the outer and inner tubes carry equal currents I in opposite directions. Calculate the inductance per unit length for such a coaxial cable.



7. (Optics) Two linear polarizers are arranged such that the angle of the transmission axis of one can be rotated with respect to the other. Randomly polarized light is incident on the two polarizers.
- (a) Plot the fraction of light that is transmitted as a function of the angle between the two transmission axes.
 - (b) If the two polarizers have their axes orthogonal to each other, and a third polarizer is placed between them midway at a 45-degree angle, what is the transmission?
 - (c) Describe the effect that having $N \gg 1$ polarizers would have such that the total angular difference is 90-degrees, but each individual polarizer is rotated by an angle of $90^\circ/N$ with respect to the previous polarizer.

8. (Thermodynamics) Consider some gas in a box of volume V , at pressure p and temperature T .

- (a) Write down or derive a “Maxwell relation” giving the rate at which the entropy of the gas changes with volume as the gas is compressed at constant temperature.
- (b) Show that the amount of heat ΔQ that flows into the gas when it is compressed by a small amount ΔV at constant temperature is given by

$$\Delta Q = T \frac{\beta_p}{\kappa_T} \Delta V,$$

where β_p and κ_T are the cubic expansivity and isothermal compressibility of the gas respectively:

$$\beta_p = \frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_p, \quad \kappa_T = -\frac{1}{V} \left(\frac{\partial V}{\partial p} \right)_T.$$

- (c) Using this expression, show if heat will in fact flow in or out of the gas when the gas is compressed.
- (d) Will the amount of heat flowing be greater or smaller if the gas is compressed adiabatically by the same amount? Why?

PART B: Optional Problems

9. (Optics) Professor Old designs an experiment to demonstrate the wave-like nature of light. The experiment consists of a plane-monochromatic light source (of wavelength λ) that illuminates four narrow slits at normal incidence. The slits are separated by increasing distances, h , $2h$ and $3h$, where $h \gg \lambda$. An observation screen is placed far from the slits such that you can neglect any difference in the observation angle (θ) from the various slits.
- (a) Derive an expression for the intensity profile (as a function of angle) when all four slits are equally illuminated.
 - (b) What is the maximum intensity and at what angles does it occur?
 - (c) What is the minimum intensity and at what angles does it occur?

10. (Mechanics) Consider a gravitational potential of the form

$$\Psi = -\frac{\Psi_0}{1 + r/r_0},$$

where Ψ_0 and r_0 are constants.

- (a) Find the corresponding mass density profile $\rho(r)$.
- (b) Find the mass enclosed within radius r , $M(r)$.
- (c) Find the total mass of the system.

11. (E&M) A very small piece of magnetized material (called a “particle” in the following), whose magnetic dipole moment M points in the x direction, moves with speed $v \ll c$ along the x -axis. A closed circular loop of thin wire with radius a and total resistance R lies in the $y - z$ plane at position $x = 0$. The motion of the particle is towards the loop.

Determine the force on the particle (both magnitude and direction). Assume that the distance of the particle from the loop is much larger than the loop radius a and the loop has negligible self-inductance.

12. (Thermodynamics) During winter when the air temperature is below the freezing temperature ($T_{air} < 0^\circ\text{C}$) ice can start forming on the surface of a lake. Assuming that the air temperature remains constant, show that the thickness of ice L will increase with time, t , according to $L \propto t^\alpha$, where α is a dimensionless constant. Determine α .