

Name: _____ Name-code: _____

University of Michigan Physics Department Graduate Qualifying Examination

Part I - Classical Physics
Saturday, January 6, 2007 9:00am-1:00pm

Please write your name-code on each page of the exam.

This is a closed book exam – but you may use the “Constants, Conversions, and Formulas” sheet we provide for this exam.

Show your work in an organized manner to receive partial for it. If you need to make an assumption or an estimate, indicate it clearly.

You must answer the first 8 obligatory questions and two of the optional four questions. Indicate which of the latter you wish us to grade (e.g., circle the question number). We will only grade the indicated optional questions.

Please do not write on this page. The hatched column to the right and the total score will be filled in by the graders.

Good Luck!

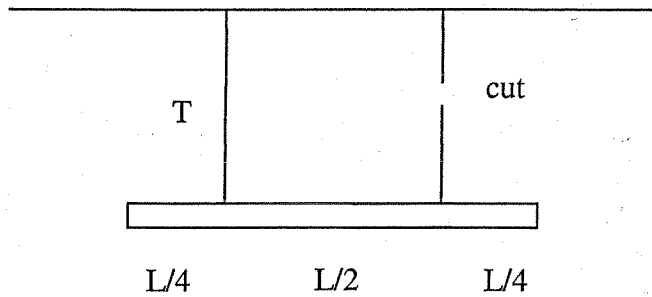
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Part A: Obligatory Problems

1. A thin, uniform rod of mass M and length L is suspended horizontally from two vertical strings attached at a distance $L/4$ from either end as shown. What is the tension T in the intact string immediately after the other string is cut?



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2. A point particle with mass M is attached to one end of massless string with spring constant K . When unstretched, the length of the string is L . The other end of the spring is attached to a fixed point but the spring can rotate freely around this point. The particle and spring are constrained to move without friction on a two-dimensional, horizontal surface containing the fixed point.
- a) If the mass moves on a circular path with radius $r = 2L$, what is the period of the motion?
- b) If the circular motion with $r = 2L$ is now slightly perturbed in the radial direction, what is the frequency of small radial oscillations?

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3. A point charge q is brought up from infinity to a point a distance a away from an isolated, uncharged conducting sphere of radius R ($a > R$). Find the potential outside the sphere assuming $V = 0$ at infinity. What is the potential of the sphere?

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4. A light beam of intensity I enters a non-conducting and non-magnetic medium at normal incidence. If the index of refraction of the medium is n , what is the radiation pressure on the medium surface?

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5. Two identical mono-atomic ideal gases have the same pressure P and the same number of particles N , but different temperatures T_1 and T_2 . They are initially confined in two vessels, of volume V_1 and V_2 , which are then connected.

Find the change in entropy after the system has reached equilibrium.

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6. Consider a van der Waals gas with the equation of state:

$$\left(P + \frac{A}{V^2}\right)(V - B) = NkT$$

where P is the pressure, V is the volume, T is the temperature, k is the Boltzmann constant, N is the number of particles. The constants A and B determine departure from the ideal gas law. Show that the general entropy change for a van der Waals gas with a constant heat capacity $C_v = 3Nk/2$ taken from (T_i, V_i) to (T_f, V_f) is

$$\Delta S = Nk \left[\frac{3}{2} \ln \left(\frac{T_f}{T_i} \right) + \ln \left(\frac{V_f - B}{V_i - B} \right) \right]$$

You will need one of the Maxwell relations:

$$\left(\frac{\partial T}{\partial V} \right)_S = - \left(\frac{\partial P}{\partial S} \right)_V, \quad \left(\frac{\partial T}{\partial P} \right)_S = \left(\frac{\partial V}{\partial S} \right)_P$$

$$\left(\frac{\partial P}{\partial T} \right)_V = \left(\frac{\partial S}{\partial V} \right)_T, \quad \left(\frac{\partial V}{\partial T} \right)_P = - \left(\frac{\partial S}{\partial P} \right)_T$$

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7. You wish to produce circularly polarized light from linearly polarized light at 589 nm. The only material available is calcite ($n_o = 1.659$, $n_e = 1.487$). (a) Calculate the thickness required to produce a true zero-order wave plate capable of producing circularly polarized light. (b) Would quartz ($n_o = 1.54424$, $n_e = 1.55336$) be better or worse for this purpose? Why?

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8. A particle of mass m moves in a circle of radius R under the influence of a central attractive force

$$F = -\frac{K}{r^2}e^{-r/a}$$

Determine the conditions on the constant a such that the circular motion will be stable.

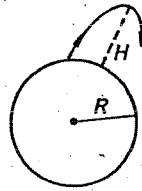
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Part B: Optional Problems

9. An object with mass M is dropped from rest and falls through air under the combined influence of the gravitational force of magnitude Mg and a frictional force opposite the direction of motion with magnitude bv^2 , where b is a constant and v is the speed of the object. How far must the object fall to reach 90% of its terminal speed? Express your answer in terms of M , g , and b .

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10. A long-range rocket is fired from the surface of the earth (radius R) with velocity $\vec{v} = (v_r, v_\theta)$, where v_r is the vertical component and v_θ the horizontal component of \vec{v} (see the figure). Neglecting air friction and the rotation of the earth (but using the exact gravitational field), obtain an equation to determine the maximum height H achieved by the trajectory. Solve it to lowest order in (H/R) and verify that it gives a familiar result for the case that \vec{v} is vertical (i.e., $v_\theta = 0$). (Hint: Be sure your solution conserves the angular momentum of the rocket, etc. You may assume the earth has a large mass relative to the rocket.)



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11. A sphere (radius R) of linear magnetic material of permeability μ is placed in an otherwise uniform magnetic field \vec{B}_0 . Find the new field inside the sphere.

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12. A nearly collimated femtosecond laser beam with peak power of 10^{10} W and a beam diameter of 2 mm is incident on a gold mirror with a damage threshold of 5×10^{10} W/cm². You have a box containing a variety of 25 mm diameter anti-reflection coated plano-convex and plano-concave lenses with focal lengths of ± 50 , ± 75 , ± 100 , ± 150 , ± 200 , and ± 250 mm. Design a telescope to ensure that the beam incident on the gold mirror is collimated and below the damage threshold. Note however that space is at a premium and you would like to make the telescope as compact as possible. How would your design change if the goal was to minimize the material dispersion in the beam path?