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University of Michigan Physics Department
Graduate Qualifying Examination

Part I - Classical Physics
Saturday, May 5, 2006 9:00am-1:00pm

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- Please write your name-code on each page of the exam.
- This is a closed book exam – but you may use the “Constants, Conversions, and Formulas” sheet we provide for this exam.
- Show your work in an organized manner to receive partial for it. If you need to make an assumption or an estimate, indicate it clearly.
- You must answer the first 8 obligatory questions and two of the optional four questions. Indicate which of the latter you wish us to grade (e.g., circle the question number). We will only grade the indicated optional questions.
- Please do not write on this page. The hatched column to the right and the total score will be filled in by the graders.
- Good Luck!
- Total Score:

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3. A conducting sphere of radius R and with charge q is half embedded in linear dielectric material of permittivity ϵ that occupies the region $z < 0$.
- (a) Determine the potential $V(r)$ in regions $z > 0$ and $z < 0$.
 - (b) Describe the charge distribution on the sphere.
 - (c) Find the bound charge distributions.

Here r is the distance from a point of interest to the center of the sphere.

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4. A spherical shell of radius R carries a surface charge distribution $\sigma(\theta) = \sigma_0 \cos \theta$. Show that the only non-vanishing moment in the multipole expansion of the electric potential is the dipole moment. Calculate the potential inside and outside the shell.

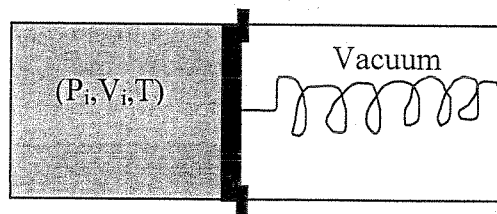
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5. Two vessels, each of volume V , are connected with a tube of length L and small cross section A ($A \ll V/L$). Initially, one vessel contains a mixture of carbon monoxide (CO) at a partial pressure P_0 and nitrogen (N_2) at a partial pressure $P - P_0$ while the other vessel contains nitrogen at a pressure P . The coefficient of diffusion of CO into N_2 or *vice versa* is D .

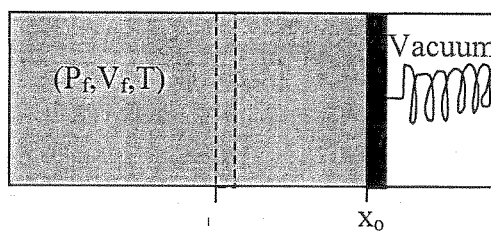
Calculate $P_0(t)$, the partial pressure CO in the first vessel as a function of time.

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6. Consider a perfect spring connected at rest (i.e., neither under compression or extension) in the right-hand chamber of the apparatus below. This side is evacuated, but an ideal gas at the initial state (P_i, V_i, T) is confined in the left chamber by a piston held in place by stops.



The spring is connected to a piston so that when the stops are removed the gas expands isothermally to the state (P_f, V_f, T) and compresses the spring a distance x_0 . Calculate the work involved in this process, first taking the gas to be the system, then taking the spring to be the system. What is the change in internal energy, ΔU , and enthalpy, ΔH , for the gas?

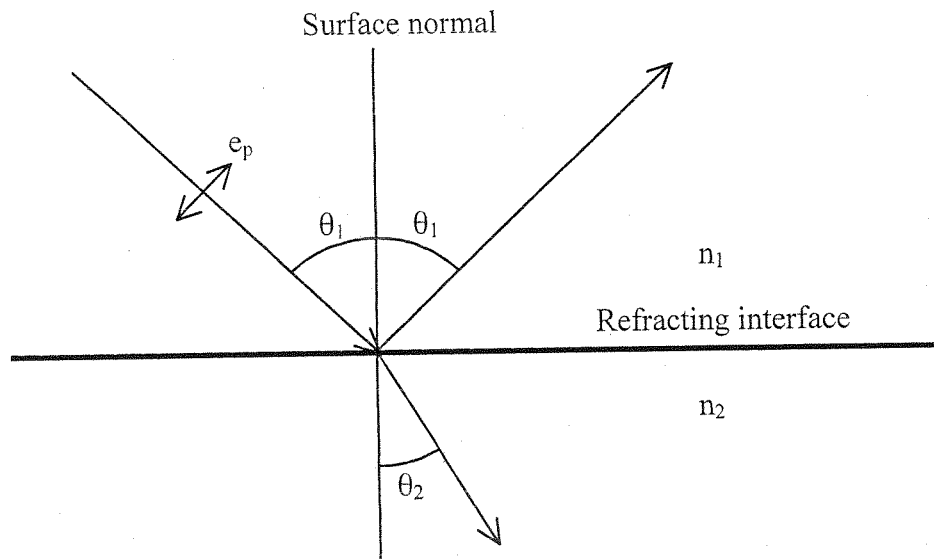


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7. In a prism compressor you would like to minimize the reflection of p-polarized 800 nm light in a BK7 prism ($n_{BK7} = 1.51078$, $n_{air} = 1.00027$). Derive a formula for Brewster's angle from the reflection factors:

$$R_p = \left(\frac{n_2 / \cos \theta_2 - n_1 / \cos \theta_1}{n_2 / \cos \theta_2 + n_1 / \cos \theta_1} \right)^2 \quad \text{and} \quad R_s = \left(\frac{n_2 \cos \theta_2 - n_1 \cos \theta_1}{n_2 \cos \theta_2 + n_1 \cos \theta_1} \right)^2$$

where:



What is your incidence angle for minimum reflection? What would be the reflection loss from four reflections if you accidentally put s-polarized radiation into the system?

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8. a) Find the central force which results in the following orbit for a particle:

$$r = a(1 + \cos \theta)$$

b) Sketch the orbit.

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Part B: Optional Problems

9. A particle with mass M is constrained to slide on a frictionless parabolic wire defined by $y = kx^2$, where k is a constant. The only forces acting on the particle are gravity with magnitude Mg directed along the negative y -axis and the force of constraint generated by the wire. The x -axis is directed horizontally.
- Give the lagrangian of the particle, using x as the independent variable.
 - Find the equation of motion for $x(t)$ using this lagrangian.
 - What is the frequency of small oscillations around the point $x = 0$?
 - In terms of the given parameters, what limit must be imposed on the amplitude of the oscillation for the answer in c) to be valid?

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10. A planet of mass m is orbiting a star of mass M . The planet experiences a small drag force $\vec{F} = -\alpha\vec{v}$ due to motion through the star's dense atmosphere. Assuming an essentially circular orbit with radius $r = r_0$ at $t = 0$, calculate the time dependence of the radius.

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11. Imagine an iron sphere of radius R that carries a charge Q and a uniform magnetization $\vec{M} = M\hat{z}$. The sphere is initially at rest.
- (a) Calculate the angular momentum stored in the electromagnetic fields.
 - (b) Suppose the sphere is gradually (and uniformly) demagnetized. Determine the induced electric field and find the torque this field exerts on the sphere. Calculate the total angular momentum imparted to the sphere in the course of the demagnetization.

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12. A nearly collimated femtosecond laser beam with a peak power of 10^{10} W and a beam diameter of 2 mm is incident on a gold mirror with a damage threshold of 5×10^{10} W/cm². You have a box containing a variety of 25 mm diameter anti-reflection coated plano-convex and plano-concave lenses with focal lengths of ± 50 , ± 75 , ± 100 , ± 150 , ± 200 , and ± 250 mm. Design a telescope to ensure that the beam incident on the gold mirror is collimated and below the damage threshold.