Name: ______________________ Name-code: ______________________

University of Michigan Physics Department Graduate Qualifying Examination

Part I - Classical Physics
Saturday, January 7, 2006 9:00am-1:00pm

Please write your name-code on each page of the exam.

This is a closed book exam – but you may use the "Constants, Conversions, and Formulas" sheet we provide for this exam.

Show your work in an organized manner to receive partial for it. If you need to make an assumption or an estimate, indicate it clearly.

You must answer the first 8 obligatory questions and two of the optional four questions. Indicate which of the latter you wish us to grade (e.g., circle the question number). We will only grade the indicated optional questions.

Please do not write on this page. The hatched column to the right and the total score will be filled in by the graders.

Good Luck!

Total Score:

1	
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Name-code: ______________________

Part A: Obligatory Problems

1. An upright cylinder with a fixed volume of water is rotating with a constant angular speed ω. Show that the curve of the water surface is a parabola.

Name-code: ____________________

2. Consider the motion of a particle of mass m in an attractive potential

$$V(r) = -Ar^{\alpha}$$

where α is an integer, positive or negative. A is a positive constant. The particle moves in an orbit close to a circular one, with radius r_0. Thus the trajectory has r oscillating around r_0. Calculate the angle θ covered by the particle during a period of oscillations and find the values of α for which this angle is comensurable with 2π. (This is a necessary but not a sufficient condition for the orbits to be closed.)

Name-code: ______________________

3. The Earth has mass $M_E = 6.0 \times 10^{27}\,g$ and a radius $R_E = 6.4 \times 10^8$ cm. (a) Find the escape speed from the surface of the Earth. Find both the analytic formula (in terms of M_E, R_E, etc.) and the numerical value in km/s. (b) Find the escape speed from the center of the planet. Assume that you can cut a small cylindrical hole through the planet that does not affect its mass profile and assume that the density is uniform. Again, find the analytic formula and the numerical value.

Name-code: ______________________

4. A point charge q is placed a distance b above an infinite grounded conducting plane. (a) Find the induced surface charge density on the plane. (b) Now assuming the point charge is moving toward the plane at a speed $v(\ll c)$, calculate the surface current density on the plane when the charge is a distance b away from the plane, ignoring any magnetic effect.

Name-code: ______________________

5. Maxwell first pointed out that light exerts radiation pressure given by $P = I/c$ (equal to the energy density of the electromagnetic wave). C is the velocity of light. What are the MKS dimensions (units) of P? (b) Calculate the intensity I needed to suspend a perfectly-absorptive, circular disk of radius b and mass m using radiation pressure to counter the force of gravity. Assume that the disk remains horizontal and that the light is normally incident on it from below.

Name-code: ____________________

6. A scuba diver begins a dive by swimming directly out from the diver's boat a distance of 50 m on the surface and then descends straight down toward the bottom. The index of refraction of water is $n = 1.3$. Assuming the water is perfectly transparent, and the diver is brightly lit by sunlight, at what depth will a friend in the boat first be able to see him under the water?

Name-code: ______________________

7. Consider a heat engine operating between a hot heat reservoir at temperature T_H and a colder (exhaust) heat reservoir at temperature T_E. Let the work W produced by the heat engine be used to run a refrigerator that uses the same heat reservoir (at temperature T_H) as its hot source, and cools a refrigeration unit at temperature T_C. Define the efficiency ϵ of the system to be the ratio of the heat removed from the refrigeration unit Δq_C to the heat used in the heat engine Δq_1, i.e.,

$$\epsilon \equiv \frac{\Delta q_C}{\Delta q_1}$$

Find the maximum efficiency of the system (write the result in terms of T_H, T_E, and T_C).

Name-code: ____________________

8. Consider a refrigerator operating in a cyclical manner. Each cycle takes heat Q_L from a reservoir kept at low temperature T_L and deposits heat Q_H in a reservoir kept at high temperature T_H. The amount of work done by an external engine to cause these heat transfers in one cycle is W. Let $\omega = \frac{Q_L}{W}$ so that ω is a measure of efficiency of the refrigerator.

   Express the maximum possible efficiency in terms of T_L and T_H.

Name-code: ______________________

Part B: Optional Problems

9. A uniform bar of mass M and length $2l$ is suspended at the end points by two identical springs of force constant k. The bar and the springs are constrained to move only in the vertical direction. Find the frequencies of small oscillations. You may neglect effects due to gravity.

Name-code: ______________________

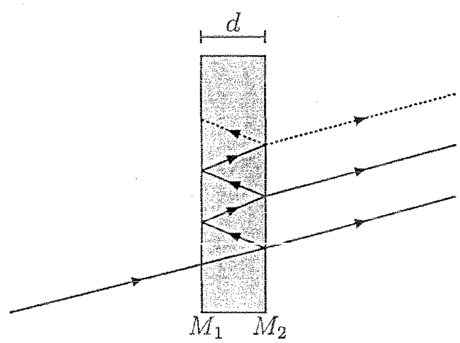
10. A small magnet of dipole moment $\vec{m}$ is levitated, with $\vec{m}$ pointing up, a distance h above a superconductor which occupies the $z < 0$ half-space. A fundamental property of a superconductor is that it "expels" magnetic fields (i.e., $\vec{B} = 0$ inside). Use the method of image, and (a) determine the location and moment of the image dipole; (b) calculate the magnetic field just above the $z = 0$ plane; (c) find the weight of the magnet.

Name-code: ______________________

11. In a 2-mirror (plane-parallel) Fabry-Perot etalon, the relative phase difference δ (in radians) between successive multiply-reflected waves is given by the optical path difference divided by the wavelength, times 2π. That is,

$$\delta = \frac{2\pi}{\lambda}(2nd\cos\theta)$$

where n is the refractive index of the medium between the mirrored interfaces M_1 and M_2, d is the mirror separation, and θ is the angle of incidence.



(a) Find the frequency separation $\nu_{m+1} - \nu_m$ of adjacent bright fringes in terms of n, d, θ, and c (the velocity of light).
(b) At normal incidence, for a fixed input wavelength and fixed n, what is the change in d that scans the device from one bright fringe to the next? That is, find $\Delta d = d_1 - d_2$ corresponding to one fringe.

Name-code: ______________________

12. Consider a small rock (an asteroid) of mass μ passing by a large planet of mass M and radius R. Assume that the asteroid has a large speed v_∞ when it is far from the planet and that the impact parameter of the interaction b is large enough that the angle of deflection is small. In this small angle limit, find an expression for the deflection angle. Find an approximate expression for the impact parameter b_0 for which the small angle approximation is expected to break down.