Name: ________________________________

University of Michigan Physics Department
Graduate Qualifying Examination

Part II - Modern Physics

Saturday, January 17, 2009 9:00am-1:00pm

This is a closed book exam - but you may use the materials provided at the exam. If you need to make an assumption or estimate, indicate it clearly. Show your work in an organized manner to receive partial credit for it.

You must answer the first 8 obligatory questions and two of the optional four questions. Indicate which of the latter you wish us to grade (e.g., circle the question number). We will only grade the indicated optional questions. Good Luck.

SOME FUNDAMENTAL CONSTANTS IN CONVENIENT UNITS

$$\text{speed of light} \quad c = 2.998 \times 10^8 \text{ m/s}$$
$$\text{electron charge} \quad e = 1.602 \times 10^{-19} \text{ C}$$
$$\text{Planck's constant} \quad h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s} = 4.136 \times 10^{-15} \text{ eV} \cdot \text{s}$$
$$\hbar = h/2\pi = 1.055 \times 10^{-34} \text{ J} \cdot \text{s} = 0.658 \times 10^{-15} \text{ eV} \cdot \text{s}$$
$$\text{Rydberg constant} \quad R_\infty = 1.097 \times 10^7 \text{ m}^{-1}$$
$$\text{Coulomb constant} \quad k = (4\pi\epsilon_0)^{-1} = 8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$$
$$\text{Universal gas constant} \quad R = 8.3 \text{ J/K} \cdot \text{mol}$$
$$\text{Avogadro's number} \quad N_A = 6 \times 10^{23} \text{ mol}^{-1}$$
$$\text{Boltzmann's constant} \quad k_B = R/N_A = 1.38 \times 10^{-23} J/K = 8.617 \times 10^{-5} \text{eV/K}$$
$$\text{Stefan} - \text{Boltzmann constant} \quad \sigma = 5.6703 \times 10^{-8} \text{ W/m}^2\text{K}^4$$
$$\text{radius of the sun} \quad R_{sun} = 6.96 \times 10^8 \text{ m}$$
$$\text{radius of the moon} \quad R_{moon} = 1.74 \times 10^6 \text{ m}$$
$$\text{radius of the earth} \quad R_{earth} = 6.37 \times 10^6 \text{ m}$$
$$\text{Gravitational constant} \quad G_N = 6.67 \times 10^{-11} m^3/kg/s^2 = 6.71 \times 10^{-39} GeV^{-2}$$

PART A: Obligatory Problems

1. (Quantum Mechanics) Eight electrons are placed in a 3-d infinite square well (cube) with each side having length $= L$. Neglect interactions between the electrons. What is the minimum energy of this system?

2. (Quantum Mechanics) Consider a particle of mass m in a one-dimensional potential well. The potential is infinite for $x < 0$ and for positive x is given by an attractive delta function "barrier" at $x = a$, i.e., $V(x) \to \infty$ for $x < 0$ and

$$V(x) = -Q\delta(x-a) \qquad \text{for} \quad x \geq 0\,.$$

Solve the Schrödinger Wave Equation for this system and find the requirement for the system to have a bound state.

3. (Quantum Mechanics) Consider a quantum system with angular momentum 1, in a state represented by the vector

$$\Psi = \frac{1}{\sqrt{26}} \begin{bmatrix} 1 \\ 4 \\ -3 \end{bmatrix}$$

Find the expectation values for the $\hat{z}$ and $\hat{x}$ components of the angular momentum, i.e., find $\langle L_z \rangle$ and $\langle L_x \rangle$.

4. (Quantum Mechanics) Consider a particle in a 3-d isotropic simple harmonic oscillator potential, $V(r) = \frac{1}{2}m\omega^2 r^2$. The energies of this particle given by $E = \hbar\omega(2n_r + l + \frac{3}{2})$, with n_r, l integers ≥ 0. Suppose the particle has spin 1/2. The generic spin-orbit coupling is given by

$$H_{SO} = \frac{1}{2m^2c^2}\mathbf{S}\cdot\mathbf{L}\frac{1}{r}\frac{dV(r)}{dr}.$$

What are the first order corrections to the energy of
(a) the ground state?
(b) the first excited state?

5. (Statistical Physics) Consider a system of N distinguishable atoms in a lattice at temperature T. Each atom has a single excited state with energy E. The ground state energy is 0. Calculate the partition function, the energy, and the specific heat for this system.

6. (Statistical Physics)

a.) For a dilute classical gas of molecules of mass M at absolute temperature T find the *most probable speed* in terms of M, T, and k_B.

b.) How will your answer change if the gas condenses to make a liquid? Will the most probable speed increase, decrease, or remain the same?

7. (Statistical Physics) Suppose that the polymers that make up a rubber band are freely jointed macromolecules as shown. Suppose that each monomer has length a, but that the total length of the chain depends on the way it folds.

You may assume that at each joint there is an equal probability to take a step to the left or the right and that bending the chain costs no energy. Figure out the entropy associated with a configuration with N_+ steps to the right and N_- to the left. Take the limit of large $N = N_+ + N_-$ with $m = N_+ - N_-$ small (i.e. $m/N << 1$). Then show that the force to extend the chain is given by $f \propto L$ where $L = a(N_+ - N_-)$ is the length. Find the constant of proportionality, i.e. the spring constant. You should assume that the internal energy, U, is independent of the length.

You might need Stirling's approximation: $\ln k! \approx k \ln k - k$ if $k >> 1$, and the fact that $\ln(1 + \delta) \approx \delta$ if $\delta << 1$

8. (Atomic Physics) Basics of the level structure of the Helium atom.

a) Sketch the energy level diagram for the lowest five (exactly five) energy levels of helium, ignoring fine structure splitting (*i.e.*, show all levels for $n = 1$ and $n = 2$). Identify each level using spectroscopic notation. The levels must be drawn in correct relative order. For each of the five levels, list the n, S and L quantum numbers in a clear, unambiguous manner (preferentially, using a well organized table).

b) What are the possible J quantum numbers for the five levels you found in a)? You may display your answer in an extra column of the table you made for part a).

c) Can you identify any metastable states? If so, which state(s) are these? Which one will have the longest lifetime?

PART B: Optional Problems

9. (Atomic Physics) The $S_{1/2}$ Rydberg levels of a certain atom in an external electric field E are characterized by a quantum defect of $\delta_s = 3.13$ and a quadratic Stark effect with a static atomic electric polarizability of

$$\alpha = 4.77 \times 10^{-41} \frac{J}{(V/m)^2} \times [n^*]^{6.5} \quad ,$$

where n^* denotes the effective quantum number. (note that $\frac{J}{(V/m)^2} = \frac{Cm^2}{V}$)

   (a) State the relationship between principal quantum number, n, effective quantum number, n^*, and the quantum defect, δ_s.

   (b) What are the energies of the states $n\,S_{1/2}$, where n denotes the principal quantum number? Assume infinite atomic mass, and express your result in wavenumbers.

   (c) A microwave field is used to drive the two-photon transition $53\,S_{1/2} \rightarrow 54\,S_{1/2}$. What is the frequency of the microwave source in GHz?

   (d) An electric field of 0.5 V/cm is applied to the atom. By how much does this change the frequency of the resonant microwave field?

10. (Nuclear Physics) The fusion of two protons into deuterium is an especially important fusion reaction because it powers our sun.
(a) Estimate the temperature at which the Coloumb barrier is overcome, and two protons can readily fuse.
(b) In the sun, fusion decay products from (a) then go on to complete additional reactions:

$$^2_1D +^1_1 H \rightarrow^3_2 He + \gamma$$
$$^3_2He +^3_2 He \rightarrow^4_2 He +^1_1 H +^1_1 H$$

When all the reactions of this chain are complete, how much energy is produced? Account for all decay products produced.

11. (Particle Physics) A neutral kaon, at rest, decays into two pions, $K^0 \rightarrow \pi^+ \, \pi^-$. The mass of the kaon is 497.6 MeV/c^2 and the mass of each pion is 139.6 MeV/c^2 The mean lifetime of a pion (in the pion rest frame) is 260 nanoseconds.

    (a) Compute the magnitude of the momentum (in MeV/c) of the π^+.

    (b) What is the speed of the π^+ (in units of c)?

    (c) In the kaon rest frame, what is the mean π^+ travel distance (in m) before the π^+ decays?

12. (Condensed Matter Physics) The carrier concentration of a metal can be determined by Hall measurements. At room temperature the Hall resistance of a 1 micron thick metal film is found to be $R_H = 1.3 \times 10^{-3}$ Ω at 1 Tesla. We will assume that there is only one type of carrier present in this metal (i.e. electrons) and the effective mass of the electrons is the same as that of free electrons.

   (a) Calculate the carrier density n at room temperature?

   (b) Calculate the Fermi wave vector k_F at room temperature?

   (c) Discuss the expected temperature dependence of the Hall resistance from 0 to 300 K.