Name: ________________________________

University of Michigan Physics Department Graduate Qualifying Examination

Part I - Classical Physics

Saturday, May 7, 2005 9:00am-1:00pm

This is a closed book exam - but you may use the materials provided at the exam. If you need to make an assumption or estimate, indicate it clearly. Show your work in an organized manner to receive partial credit for it.

You must answer the first 8 obligatory questions and two of the optional four questions. Indicate which of the latter you wish us to grade (e.g., circle the question number). We will only grade the indicated optional questions. Good Luck.

SOME FUNDAMENTAL CONSTANTS IN CONVENIENT UNITS

$$
\begin{aligned}
\text{speed of light} \quad c &= 2.998\times10^{8}\ \mathrm{m/s}\\
\text{electron charge} \quad e &= 1.602\times10^{-19}\ \mathrm{C}\\
\text{Planck's constant} \quad h &= 6.626\times10^{-34}\ \mathrm{J\cdot s} = 4.136\times10^{-15}\ \mathrm{eV\cdot c}\\
\hbar = h/2\pi &= 1.055\times10^{-34}\ \mathrm{J\cdot s} = 0.658\times10^{-15}\ \mathrm{eV\cdot s}\\
\text{Rydberg constant} \quad R_\infty &= 1.097\times10^{6}\ \mathrm{m^{-1}}\\
\text{Coulomb constant} \quad k=(4\pi\epsilon_0)^{-1} &= 8.988\times10^{9}\ \mathrm{N\cdot m^2/C^2}\\
\text{Universal gas constant} \quad R &= 8.3\mathrm{J/K\cdot mol}\\
\text{Avogadro's number} \quad N_A &= 6\times10^{23}\ \mathrm{mol^{-1}}\\
\text{Boltzmann's constant} \quad k_B = R/N_A &= 1.38\times10^{-23}J/K = 8.617\times10^{-5}\mathrm{eV/K}\\
\text{Stefan – Boltzmann constant} \quad \sigma &= 5.6703\times10^{-8}\ \mathrm{W/m^2K^4}\\
\text{radius of the sun} \quad R_{sun} &= 6.96\times10^{8}\ \mathrm{m}\\
\text{radius of the moon} \quad R_{moon} &= 1.74\times10^{6}\ \mathrm{m}\\
\text{radius of the earth} \quad R_{earth} &= 6.37\times10^{6}\ \mathrm{m}\\
G_N &= 6.67\times10^{-11}m^3/kg/s^2 = 6.71\times10^{-39}\mathrm{GeV^{-2}}
\end{aligned}
$$

SOME USEFUL CONVERSIONS AND COMBINATIONS

$$
\begin{aligned}
\text{fine structure constant} \quad \alpha &= ke^2/\hbar c = 1/137\\
\text{Bohr magneton} \quad e\hbar/2m_e &= 9.27\times10^{-24}\mathrm{J/T} = 5.79\times10^{-5}\mathrm{eV/T}\\
hc &= 19.865\times10^{-26}\ \mathrm{J\cdot m} = 12.41\times10^{3}\ \mathrm{eV\cdot \mathring{A}} = 1241\ \mathrm{MeV\cdot fm}\\
\hbar c &= 3.165\times10^{-26}\ \mathrm{J\cdot m} = 1973\ \mathrm{eV\cdot \mathring{A}} = 197.3\ \mathrm{MeV\cdot fm}\\
ke^2 &= 1.44\ \mathrm{MeV\cdot fm}
\end{aligned}
$$

$1\mathrm{\mathring{A}} = 10^{-10}\ \mathrm{m} = 10^{5}\ \mathrm{fm}$ $\qquad$ $1\mathrm{eV} = 1.602\times10^{-19}\ \mathrm{J}$

SOME USEFUL RELATIONS

$$\frac{C_V}{Nk} = 9\,(T/\theta_D)^3 \int_0^{\theta_D/T} \frac{x^4 e^4 dx}{(e^x-1)^2} \quad \text{(Debye formula)}$$

$$= 3\left[1 - \frac{1}{20}\left(\frac{\theta_D}{T}\right)^2 + \ldots\right] = \frac{12\pi^4}{5}\left(\frac{T}{\theta_D}\right)^3 (1+\ldots)$$

$$\frac{U}{N} = \frac{3}{5}E_F\left[1 + \frac{5\pi^2}{12}\left(\frac{kT}{E_F}\right)^2 - \frac{\pi^4}{16}\left(\frac{kT}{E_F}\right)^4 + \ldots\right] \quad \text{(degenerate electron gas)}$$

$$E_F = \frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{2/3} \quad \text{(nonrelativistic Fermi energy)}$$

$$r = r_\circ A^{1/3}, \qquad r_\circ = 1.2\times 10^{-15} m \quad \text{(approximate average nuclear radius)}$$

$$E_b = C_1 A - C_2 A^{2/3} - C_3\frac{Z(Z-1)}{A^{1/3}} - C_4\frac{(N-Z)^2}{A} \quad \text{(semiempirical binding energy of a nucleus)}$$

$$\frac{n}{s} + \frac{n\prime}{s\prime} = \frac{n\prime - n}{R} \text{(refraction of paraxial rays)}$$

MASSES OF SOME ELEMENTARY PARTICLES

	Rest Mass, m_0 (kg)	m_0c^2 (MeV)
Electron	9.109×10^{-31}	0.511
Proton	1.673×10^{-27}	938.3
bNeutron	1.675×10^{-27}	939.6
Atomic mass unit (1 amu)	1.661×10^{-27}	931.5

VISIBLE LIGHT SPECTRUM

300	400	500		600		700(Nanometer)	
← Ultraviolet	Violet	Blue	Green	Yellow	Orange	Red	Infrared →

Part A: Obligatory Problems

1. A particle of mass m is attached to a spring of force constant k. The system is streched a distance L and released from rest.

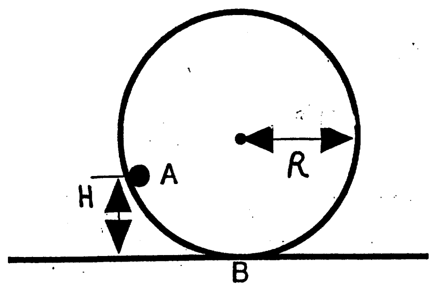
   (a) What is the maximum speed of the particle and where does it occur?

   (b) What is the maximum acceleration of the particle and where does it occur?

2. A steel ball of radius r is rolling on a frictionless vertical circular track. Its rotational inertia about the center of mass is $I = amr^2$, where r is the radius of the ball. If the ball is released from rest at point A,

   (a) What is its speed at point B at the bottom of the track?

   (b) What is the force of the track on the ball at point B and in which direction does it act?



3. The FRW equations

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho$$
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

control the evolution of homogeneous cosmologies with scale factor $a(t)$ and flat spatial sections. a) Assuming $p = w\rho$, where w is a constant, determine the dependence of the energy density ρ on the scale factor for radiation dominated ($w = 1/3$) , matter dominated ($w = 0$), and dark energy dominated ($w = -1$) cosmologies. b) Find the time-dependence of the scale factor for each of the three equations of state.

4. A non-relativistic electron with an initial velocity $\vec{v}_0$ is perpendicularly injected into a uniform and static magnetic field $\vec{B}$, *i.e.* $\vec{v}_0 \perp \vec{B}$. Find the electron's speed as a function of time as the result of radiative energy loss.

   Neglect the tangential radiation and express your results using e, m, c, v_0 and B. You will need Larmor's formula for energy loss

$$\frac{dE}{dt} = -\frac{2}{3}\frac{e^2}{c^3}|\vec{a}|^2.$$

5. Consider a space which is half-filled with linear dielectric material with permittivity ϵ and permeability μ. A right-travelling electromagnetic plane wave of angular frequency ω encounters the material surface at normal incidence. Derive the reflection coefficient using boundary conditions. Express your result using μ_0, ϵ_0, μ and ϵ.

6. A point electric dipole moment $\vec{p}$ is located at the center of a spherical hole of radius a inside a dielectric medium of infinite extent and dielectric constant ϵ. Find the electric field everywhere and bound charge density on the surface of the dielectric.

7. **Temperature change on compression of a gas:**

(a) Show that the rate at which the temperature changes when you compress a gas adiabatically is

$$\left(\frac{\partial T}{\partial V}\right)_S = -\frac{T}{C_V}\left(\frac{\partial p}{\partial T}\right)_V,$$

where p, V, T and S are the pressure, volume, temperature and entropy, and C_V is the heat capacity at constant volume.

(b) A particular sample of gas in a box obeys the equation of state $p(V - b) = RT$, where b and R are constants. Assuming that the heat capacity C_V is constant over the temperature range of interest, show that upon adiabatic compression from an initial volume and temperature V_1, T_1 to a final volume V_2, the temperature of the gas rises to

$$T_2 = T_1 \left[\frac{V_1 - b}{V_2 - b}\right]^{R/C_V}$$

8. Consider diffraction of plane monochromatic light through two vertical slits of width b separated by a distance $h = 3b$, where $b \gg \lambda$. Describe the diffraction pattern in the Fraunhofer limit. At what angles do you find the bright and dark fringes?

Part B: Optional Problems

9. **Thermodynamics of a spring:** A Hooke's law (i.e., linear) spring as pictured has a spring constant and resting length that depend on temperature, so that we need to use thermodynamics to calculate its behavior. A small amount of work done on the spring by stretching it is $dW = f dL$, where f and L are the force on the spring and its length.

   (a) Write down an expression for a small change dF in the Helmholtz free energy of the spring in terms of temperature T, entropy S, and f and L. From this derive a Maxwell relation giving $(\partial S/\partial L)_T$ in terms of T, f, and L.

   (b) The equation of state of the spring is

   $$f = a\frac{L}{T} - b,$$

   where a and b are constants. Find the amount of heat Q that flows into the spring when we isothermally stretch it from length L_1 to length L_2.

10. A collimated TEM_{00} Gaussian laser beam has a cylindrically symmetric intensity (irradiance) profile

$$I(r,t) = I_0(t)e^{-2r^2/\omega^2} \quad (\omega = 2\sigma)$$

and a wavelength $\lambda = 1\,\mu m$. If the peak intensity of the laser is $10\,\mathrm{GW/cm^2}$ for a width of $\omega = 1\,\mathrm{cm}$, what would the width and peak intensity be at the focus of a 30 cm lens (of sufficient diameter that there are negligible diffraction losses)? Compare the peak electric field seen at the focus, with the electric field seen by an electron orbiting in the ground state of a hydrogen atom.

11. Suppose a particle in 1d is subjected to a potential $a/\cos^2(x/X)$. Using action-angle variables,

   (a) set up an expression for J, but do not do the integral. For what E's is it possible to use action angle variables? Sketch the potential near the origin.

   (b) Look at the case of small oscillations, $x \ll X$. Get the frequency by elementary means.

   (c) Do the integral for dJ/dE for the case of small oscillations, and compare to the answer in (b). You may need the integral

$$\int \frac{dx}{(u^2 - x^2)^{1/2}} = \sin^{-1}(x/u)$$

12. If we lie down on a surface the surface pushes back up on us, and we feel that pressure, which is just sufficient to hold our body up. Why is it more comfortable to lie on a bed than on a flat piece of wood? Estimate the relevant pressures.