Name: ____________________

University of Michigan Physics Department Graduate Qualifying Examination

Part II - Modern Physics

Saturday, May 14, 2005 9:00am-1:00pm

This is a closed book exam - but you may use the materials provided at the exam. If you need to make an assumption or estimate, indicate it clearly. Show your work in an organized manner to receive partial credit for it.

You must answer the first 8 obligatory questions and two of the optional four questions. Indicate which of the latter you wish us to grade (e.g., circle the question number). We will only grade the indicated optional questions. Good Luck.

SOME FUNDAMENTAL CONSTANTS IN CONVENIENT UNITS

$$
\begin{aligned}
\text{speed of light} \quad c &= 2.998\times 10^{8}\ \mathrm{m/s} \\
\text{electron charge} \quad e &= 1.602\times 10^{-19}\ \mathrm{C} \\
\text{Planck's constant} \quad h &= 6.626\times 10^{-34}\ \mathrm{J\cdot s} = 4.136\times 10^{-15}\ \mathrm{eV\cdot c} \\
\hbar = h/2\pi &= 1.055\times 10^{-34}\ \mathrm{J\cdot s} = 0.658\times 10^{-15}\ \mathrm{eV\cdot s} \\
\text{Rydberg constant} \quad R_\infty &= 1.097\times 10^{6}\ \mathrm{m^{-1}} \\
\text{Coulomb constant} \quad k = (4\pi\epsilon_0)^{-1} &= 8.988\times 10^{9}\ \mathrm{N\cdot m^2/C^2} \\
\text{Universal gas constant} \quad R &= 8.3\mathrm{J/K\cdot mol} \\
\text{Avogadro's number} \quad N_A &= 6\times 10^{23}\ \mathrm{mol^{-1}} \\
\text{Boltzmann's constant} \quad k_B = R/N_A &= 1.38\times 10^{-23} J/K = 8.617\times 10^{-5}\mathrm{eV/K} \\
\text{Stefan - Boltzmann constant} \quad \sigma &= 5.6703\times 10^{-8}\ \mathrm{W/m^2K^4} \\
\text{radius of the sun} \quad R_{sun} &= 6.96\times 10^{8}\ \mathrm{m} \\
\text{radius of the moon} \quad R_{moon} &= 1.74\times 10^{6}\ \mathrm{m} \\
\text{radius of the earth} \quad R_{earth} &= 6.37\times 10^{6}\ \mathrm{m} \\
G_N &= 6.67\times 10^{-11} m^3/kg/s^2 = 6.71\times 10^{-39} GeV^{-2}
\end{aligned}
$$

SOME USEFUL CONVERSIONS AND COMBINATIONS

$$
\begin{aligned}
\text{fine structure constant} \quad \alpha &= ke^2/\hbar c = 1/137 \\
\text{Bohr magneton} \quad e\hbar/2m_e &= 9.27\times 10^{-24}\mathrm{J/T} = 5.79\times 10^{-5}\mathrm{eV/T} \\
hc &= 19.865\times 10^{-26}\ \mathrm{J\cdot m} = 12.41\times 10^{3}\ \mathrm{eV\cdot \mathring{A}} = 1241\ \mathrm{MeV\cdot fm} \\
\hbar c &= 3.165\times 10^{-26}\ \mathrm{J\cdot m} = 1973\ \mathrm{eV\cdot \mathring{A}} = 197.3\ \mathrm{MeV\cdot fm} \\
ke^2 &= 1.44\ \mathrm{MeV\cdot fm} \\
& 1\mathrm{\mathring{A}} = 10^{-10}\ \mathrm{m} = 10^{5}\ \mathrm{fm} \qquad 1\mathrm{eV} = 1.602\times 10^{-19}\ \mathrm{J}
\end{aligned}
$$

SOME USEFUL RELATIONS

$$\frac{C_V}{Nk} = 9\,(T/\theta_D)^3 \int_0^{\theta_D/T} \frac{x^4 e^4 dx}{(e^x-1)^2} \quad \text{(Debye formula)}$$

$$= 3\left[1 - \frac{1}{20}\left(\frac{\theta_D}{T}\right)^2 + \ldots\right] = \frac{12\pi^4}{5}\left(\frac{T}{\theta_D}\right)^3 (1+\ldots)$$

$$\frac{U}{N} = \frac{3}{5}E_F\left[1 + \frac{5\pi^2}{12}\left(\frac{kT}{E_F}\right)^2 - \frac{\pi^4}{16}\left(\frac{kT}{E_F}\right)^4 + \ldots\right] \quad \text{(degenerate electron gas)}$$

$$E_F = \frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{2/3} \quad \text{(nonrelativistic Fermi energy)}$$

$$r = r_\circ A^{1/3}, \qquad r_\circ = 1.2 \times 10^{-15} m \quad \text{(approximate average nuclear radius)}$$

$$E_b = C_1 A - C_2 A^{2/3} - C_3 \frac{Z(Z-1)}{A^{1/3}} - C_4 \frac{(N-Z)^2}{A} \quad \text{(semiempirical binding energy of a nucleus)}$$

$$\frac{n}{s} + \frac{n\prime}{s\prime} = \frac{n\prime - n}{R} \text{(refraction of paraxial rays)}$$

MASSES OF SOME ELEMENTARY PARTICLES

	Rest Mass, m_0 (kg)	m_0c^2 (MeV)
Electron	9.109×10^{-31}	0.511
Proton	1.673×10^{-27}	938.3
bNeutron	1.675×10^{-27}	939.6
Atomic mass unit (1 amu)	1.661×10^{-27}	931.5

VISIBLE LIGHT SPECTRUM

300	400	500		600		700(Nanometer)	
← Ultraviolet	Violet	Blue	Green	Yellow	Orange	Red	Infrared →

Part A: Obligatory Problems

1. Imagine a system in which there are just two linearly independent states,

$$|1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad and \quad |2\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

If the Hamiltonian matrix of the system has the form

$$H = \hbar \begin{pmatrix} \omega & \omega' \\ \omega' & \omega \end{pmatrix}$$

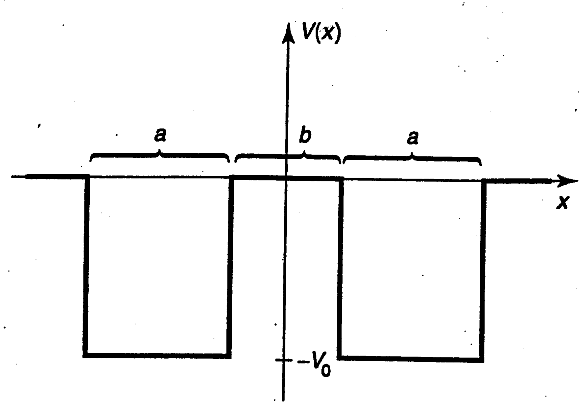
where ω and ω' are real constants, find the probability that the system remains in state $|1\rangle$ at a late time t if it starts out (at $t = 0$) in state $|1\rangle$.

2. (Note: This problem does not require detailed calculations.) Consider the "double square well" potential in the figure. Suppose the depth V_0 and the width a are fixed, and great enough so that several bound states occur.

   (a) Sketch the ground-state wave function ψ_1 and the first excited state ψ_2, (i) for the case $b \simeq 0$, (ii) for $b \simeq a$, and (iii) for $b \gg a$.

   (b) Qualitatively, how do the corresponding energies (E_1 and E_2) vary, as b goes from 0 to ∞?

   (c) The double well models the potential an electron experiences in a diatomic molecule (the two wells represent the attractive force of the nuclei). If the nuclei are free to move, they will adopt the configuration of minimum energy. In view of your conclusions in (b), does the electron tend to draw the nuclei together, or push them apart? Explain. (Do not worry about the internuclear repulsion.)



3. Suppose we put a delta-function bump in the center of the infinite square well:

$$H' = \alpha\delta(x - a/2)$$

where α is a constant. Using perturbation theory technique, find the first-order correction to the allowed energies. Some energy levels are not perturbed. Why?

4. Estimate the force (in Newtons) on a proton near the surface of a $^{238}_{92}U$ nucleus. ($\epsilon_0 = 8.85 \times 10^{-12}\, F/m$ in SI units.)

5. In a Heisenberg magnet there is an excitation called a *spin wave* or *magnon*. Suppose that the spin wave has dispersion $\omega(\vec{k}) = Dk^2$ for N normal modes, and that spin waves are (massless) bosons. Each spin wave reduces the magnetic moment, $\mathcal{M}$, by removing one quantum of angular momentum, i.e., $\mathcal{M}(T) = \mathcal{M}(T=0) - \mu\langle n\rangle$, where μ is the magnetic moment per spin and $\langle n\rangle$ the mean number of magnons. Find the low temperature behavior of $\mathcal{M}(T)$ in 3d.

6. Consider the $2p$ state in hydrogen.

   (a) List all of the states in the j, m_j basis.

   (b) Write each wavefunction as appropriate linear combinations of l, m_l, s, m_s states.

   (c) Sketch the energy levels showing the Zeeman splittings as a function of magnetic field, making sure to show the transition between low and high magnetic field.

7. The coefficient of thermal conductivity κ of a fluid is defined by

$$\vec{Q} = -\kappa \vec{\nabla} T$$

Here $\vec{Q}$ is the heat flux (heat through an area per unit time and unit area) and T is the temperature. Estimate κ.

Write your result in terms of the specific heat c, the mean velocity of molecules in the fluid $\bar{v}$, and the interaction cross-section σ_0.

8. Consider the Fermi distribution function:

$$f(\epsilon) = \frac{1}{\exp\beta(\epsilon - \mu) + 1}$$

Here, μ is the chemical potential which is determined by the equation (for electrons with $s = 1/2$):

$$N = 2\sum_{\vec{k}} f(\epsilon_{\vec{k}})$$

Figure out an exact expression for $\mu(T)$ in two dimensions. *Hint: You will need the integral*

$$\int_0^\infty \frac{dy}{ae^y + 1} = \ln(a+1) - \ln(a)$$

Part B: Optional Problems

9. Suppose it is known that charged pions decay into $\mu\nu$ and into $e\nu$. Suggest an experiment that could prove or disprove the idea that the neutrinos in these decays are different.

10. **Phonon modes:**

(a) Derive an expression for the density $n(\omega)$ of phonon modes in a (three dimensional) classical continuous elastic medium in terms of the volume V and the speed of sound v (which can be assumed constant).

(b) In a non-continuous medium – one made out of atoms – the total number of degrees of freedom is $3N$, where N is the number of atoms. Derive a formula for the Debye frequency of such a medium.

11. A tritium atom (^3H) decays into a $^3He^+$ ion. What is the probability that the $^3He^+$ ion from this decay is in its ground state?

12. In the vicinity of a second order phase transition the specific heat contains a contribution of the singular form

$$C_p \propto |t|^{-\alpha} \ : \quad -1 < \alpha < 0$$

where $t = T - T_c$. As the phase transition is reached the order parameter η behaves as

$$\eta \propto (-t)^{\beta} \ ; \ \beta > 0$$

The susceptibility of an external field h will be

$$\chi \propto |t|^{-\gamma} \ ; \ \gamma > 0$$

It is understood that the external field is weak, such that the induced order parameter $\eta_{\text{ind}} = \chi h$ remains below the spontaneous order parameter η_{sp} in the broken phase. Derive a relation between the critical exponents α, β and γ.