Minimum Bias Measurements at ATLAS on the LHC at 900 GeV

by

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A senior thesis submitted in partial fulfillment of the requirements for the degree of Honors Bachelor of Science (Physics) in The University of Michigan 2010
ACKNOWLEDGEMENTS

I would like to thank Professor Jianming Qian for taking me on and allowing me to work for him over the past couple years. Further, I’d like to thank the Michigan Higgs group, particularly Devin Harper, for their willingness to answer my various questions on both physics and computing.
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LIST OF ABBREVIATIONS

**ATLAS**  A Toroidal LHC ApparatuS

**LHC**  Large Hadron Collider

**SCT**  Semiconductor Tracker

**TRT**  Transition Radiation Tracker

**TGCs**  Thin Gap Chambers

**RPCs**  Resistive Plate Chambers

**MDTs**  Monitored Drift Tubes

**CSCs**  Cathode Strip Chambers

**CMS**  Compact Muon Solenoid

**ALICE**  A Large Ion Collider Experiment

**LHCb**  Large Hadron Collider beauty

**LHCf**  Large Hadron Collider forward

**TOTEM**  TOTal Elastic and diffractive cross section Measurement

**ID**  Inner Detector

**MBTS**  Minimum Bias Trigger Scintillators
Presented is a measurement of the value $\frac{1}{N_{ev}} \frac{dN_{ch}}{d\eta}$ in p-p (proton-proton) collisions at 900 GeV center-of-mass energy at the Large Hadron Collider (LHC) with the ATLAS detector including a data driven correction for primary vertex reconstruction efficiency. This was found to be $1.146 \pm .141 (stat.)$ for $|\eta| < 2.5$. The distribution for track multiplicity per event, $\frac{1}{N_{ev}} \frac{dN_{ev}}{dN_{ch}}$, is also presented along with kinematic distributions for comparison of data with Monte Carlo generated events.
CHAPTER I

Introduction

This is a particularly good time to become involved in high energy physics now that data taking has begun at the LHC. It will be exciting to see what kind of results come out. Whether or not theories are confirmed, the LHC should advance our understanding of particle physics by either confirming what has almost become expected, for instance a Higgs boson or supersymmetric partner (SUSY) of a standard model particle, or showing that the predictions are wrong. The Standard Model is incomplete thus far, so no matter what, there will be something new to investigate. Though at first, efforts will be directed toward understanding the detectors and rediscovering the standard model as we know it.

Originally, I planned to write on the discovery potential of a standard model like neutral vector boson $Z'$. However, I switched topics to $dN_{ch}/d\eta$ in minimum bias events in order to get my hands on real data, rather than continue using solely Monte Carlo generated events. The data at 900 GeV was taken using the minimum bias triggers installed on ATLAS. As the name implies, the idea behind minimum bias events is to select events without introducing a bias from the selection, such as requiring the event to contain leptons which have a large amount of momentum in the transverse plane, referred to as $P_T$. Ideally, this would be done by a random trigger. However, there are only two bunches per beam and many of the bunch crossings do
not have interactions. Thus, a random trigger would pick up many empty events, which would be a waste of resources. Essentially, the minimum bias trigger should select events where something has happened.

Single diffractive and double diffractive events are those in which one or two of the protons did not break up in the collision. This is different than nominal case where both protons do not remain intact. For the purposes of this analysis, these events are uninteresting and the minimum bias trigger is designed to reject these events.
CHAPTER II

The Detector and Accelerator

The LHC is a p-p collider 27 km in circumference located at the CERN complex in Switzerland. Optimally, the LHC is designed to run with a 14 TeV center-of-mass energy with an instantaneous luminosity of $10^{34} \text{ cm}^{-2}\text{s}^{-1}$ \cite{7}. At full luminosity the bunch crossing rate will be 40 MHz \cite{6}. LHC parameters are listed in Table 2.1.

There are six experiments that will run off of the LHC: A Large Ion Collider Experiment (ALICE), ATLAS, Compact Muon Solenoid (CMS), Large Hadron Collider beauty (LHCb), Large Hadron Collider forward (LHCf), and TOTal Elastic and diffractive cross section Measurement (TOTEM). ATLAS and CMS are both general-purpose detectors designed to cover a wide range of physics, but differ in design. ALICE will study quark-gluon plasmas from lead-ion collisions. LHCb will look at the b-quark with regard to understanding the matter and anti-matter asymmetry. TOTEM will measure the proton cross-section and LHCf will look at particles produced in collisions close to the beam line \cite{7}. The LHC is currently running and taking data at 7 TeV center-of-mass energy.
### Table 2.1: LHC parameters [8].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circumference</td>
<td>26.659 km</td>
</tr>
<tr>
<td>Magnet Operating Temperature</td>
<td>1.9 K</td>
</tr>
<tr>
<td>Proton Injection Energy</td>
<td>0.45 TeV</td>
</tr>
<tr>
<td>Proton Collision Energy</td>
<td>7 TeV</td>
</tr>
<tr>
<td>Bunch Crossing Rate</td>
<td>40 MHz</td>
</tr>
<tr>
<td>Collisions per Crossing</td>
<td>$6 \times 10^8$ s$^{-1}$</td>
</tr>
<tr>
<td>Design Luminosity</td>
<td>$10^{34}$ cm$^{-2}$ s$^{-1}$</td>
</tr>
</tbody>
</table>

2.1 **The ATLAS Detector**

ATLAS is a general-purpose detector installed on the LHC. ATLAS is approximately 25 m in diameter and 45 m long, weighing about 7,000 tonnes. The detector is made up of four main parts: the Inner Detector (ID), the calorimeters, the muon system, and the magnet systems. The ID is comprised of the pixel detector, the Semiconductor Tracker (SCT), and the Transition Radiation Tracker (TRT). These are responsible for tracking electromagnetically charged particles. A strong magnetic field in the inner detector allows for the measurement of particle momentum through the bending of tracks. A liquid Argon calorimeter and tile calorimeter make up the calorimeter system. Calorimeters measure the energy of particles, both charged and neutral, by stopping the particle and measuring the energy deposited in the detector. The muon system is comprised of the Thin Gap Chambers (TGCs), Resistive Plate Chambers (RPCs), Monitored Drift Tubes (MDTs), and Cathode Strip Chambers (CSCs). The muon chambers provide tracking for muons, which pass through calorimeters. Neutrinos are not measured directly and show up as missing transverse energy when the transverse momentum of an event is summed [4]. ATLAS makes use of a solenoid magnet around the inner detector which produces a 2 T field, a barrel toroid system to produce a 0.5 T field, and two end-cap toroids which produce a 4 T field [3].

The interaction point is taken to be the origin on the coordinate system used when describing ATLAS events. The $z$-axis lies along the beam line and the $x$ and $y$ axes
Figure 2.1: The ATLAS detector and various components from Ref. [3].

form the transverse plane [3]. The x-axis points inward on the LHC ring and the y-axis is aligned vertically. The azimuthal angle $\phi = 0$ corresponds to the positive x-axis and increases toward the vertical y-axis. The polar angle $\theta$ is measured from the z-axis. Pseudorapidity is defined as $\eta = -\ln(\tan(\theta/2))$ and is used instead of $\theta$ because particle production is roughly constant as a function of pseudorapidity and $\Delta \eta$ is invariant under Lorentz boosts along the beam axis [6]. The transverse momentum $P_T$ is defined as the momentum in the x-y plane.

2.2 Inner Detector

The ID is responsible for the tracking of charged particles and vertex reconstruction. The ID covers a range of $|\eta| < 2.5$, but the TRT only covers $|\eta| < 2$. The pixel detector sits closest to the beam pipe and provides information for vertex reconstruction [3]. It is accurate to 10 $\mu$m in R-$\phi$ and 115 $\mu$m in $z$ in the barrel. The SCT surrounds the pixel detector using stereo strips in the barrel to measure R-$\phi$, and a
combination of radial and stereo strips in the end cap. The SCT has an accuracy of 17 $\mu$m in R-$\phi$ and 580 $\mu$m in $z$ in both the barrel and end cap. The TRT is made of interleaved 4 mm straw tubes and provides a large number of hits for measurement in R-$\phi$ only, with an accuracy of 130 $\mu$m [1]. The accuracy of the measurements in the R-$\phi$ plane is important because charged particles bend around the beam pipe in the solenoid field and thus this accuracy is related to measuring the particles momentum.

2.3 Calorimeters

Though not as important for this study, the calorimeters are briefly discussed. The calorimeters are responsible for measuring the energy of both charged and neutral particles in an event. Most calorimeters are constructed with alternating layers of absorbing material, which create a showers of particles, and scintillating layers, to pick up the energy.

The liquid-argon electromagnetic calorimeter is designed to measure the energy of electromagnetically interacting particles and is closet to the beam pipe. It is a lead-Liquid-Argon detector. Including both end-cap and barrel portions, this calorimeter covers $|\eta| < 3.2$.

The hadronic tile calorimeter sits outside of the electromagnetic calorimeter. Including all components, it covers a range of $|\eta| < 1.7$. The tile calorimeter uses steel as an absorber. There are also two more liquid-argon calorimeters, in the end cap and another in the forward region. This extends the coverage to greater than $|\eta| = 3.2$ [3].

2.4 Muon System

The muon system is not relevant to this study and so is only covered briefly. The MDTs cover $|\eta| < 2.7$ and serve to provide tracking data and the CSCs provide additional tracking data in the range $2.0 < |\eta| < 2.7$. The RPCs provide infor-
mation for triggering in $|\eta| < 1.05$ and the TGCs provide triggering information in $1.05 < |\eta| < 2.4$ [3].

2.5 Triggers

The Minimum Bias Trigger Scintillators (MBTS) are to select elastic interactions during low luminosity runs. Normally, a random trigger would be used in order to reduce bias, but with low luminosity the efficiency for a random trigger is very low. The job of the MBTS is to reject empty events as well as beam gas and halo events. After 3-4 months at high luminosity, the scintillators will degrade due to radiation damage.

There are 16 counters per side covering a range in the forward range of $2.1 < \eta < 3.8$. The MBTS are mounted between the ID and the Liquid-Argon-Calorimeter and the signals are read out through the calorimeter electronics [5].

The trigger efficiency was flat and nearly unity across the $\eta$ region of interest. This was determined by using an independent data sample and a control trigger in Ref. [2]. This was also presented in a study on the triggers from 2008 in Ref. [5]. The control trigger required more than 6 Pixel clusters and 6 SCT hits at L2 and one or more tracks with transverse momentum greater than 200 MeV at the event filter [2]. Therefore, no correction to the $\eta$ distribution due to the trigger is necessary.
CHAPTER III

Event and Track Selection

For this analysis, the 900 GeV data from 2009 with the solenoid and toroid magnets on as well as stable running conditions were used. This selection is represented in the good lumiblock cut. Events without any reconstructed tracks were rejected. In addition, an OR between the MBTS triggers was used at the event level. This includes L1_MBTS_1, L1_MBTS_2, and L1_MBTS_1.1. There was also a requirement of a reconstructed primary vertex with at least 3 associated tracks in the event.

Tracks were selected by requiring a \( \text{\textit{P}}_T > 500 \text{ MeV} \) to reduce problems caused by interaction with detector material. Further, the tracks were required to be associated with a valid primary vertex. This analysis was limited to \(|\eta| < 2.5\) by the coverage of the ID. The B layer in the pixel detector is closest to the interaction point. The requirement of at least one B layer hit along with the \( d_0 \) and \( z_0 \sin(\theta) \) cuts are intended to reduce contamination from secondary particles. The parameter \( d_0 \) is the impact parameter in the transverse plane of the track with respect to the beam axis. The product \( z_0 \sin(\theta) \) forms the track impact parameter along the beam axis with respect to the origin. In order to reduce backgrounds, requirements on SCT and Pixel hits were set in an attempt reject low quality tracks. The event and track selections are very close to those used in the ATLAS collaboration paper on minimum bias [2]. A table comparing relative efficiencies of the cuts in data with Monte Carlo can be found.
The Monte Carlo samples were generated with Pythia [9]. The samples mc09_900GeV.105001.pythia_minbias.recon.MTree.e500_s655_s657_d257_r1023 were used.

<table>
<thead>
<tr>
<th>Cut</th>
<th>Data</th>
<th>Rel. % Data</th>
<th>MC</th>
<th>Rel. % MC</th>
</tr>
</thead>
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<tr>
<td>Total Events</td>
<td>432336</td>
<td></td>
<td>7199639</td>
<td></td>
</tr>
<tr>
<td>Event:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N_{Tracks} \geq 1$</td>
<td>422139</td>
<td>0.976414</td>
<td>7157812</td>
<td>0.99419</td>
</tr>
<tr>
<td>MBTS* Trigger</td>
<td>422128</td>
<td>0.999974</td>
<td>7157233</td>
<td>0.999919</td>
</tr>
<tr>
<td>Good LumiBlock</td>
<td>253411</td>
<td>0.600318</td>
<td>7157233</td>
<td>1</td>
</tr>
<tr>
<td>PV $N_{Tracks} \geq 3$</td>
<td>217600</td>
<td>0.858684</td>
<td>6543539</td>
<td>0.914255</td>
</tr>
<tr>
<td>Tracks:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Valid Primary Vertex</td>
<td>217600</td>
<td>1</td>
<td>6543539</td>
<td>1</td>
</tr>
<tr>
<td>$P_T &gt; 500\text{MeV}$</td>
<td>216617</td>
<td>0.995483</td>
<td>6528801</td>
<td>0.997748</td>
</tr>
<tr>
<td>$</td>
<td>\eta</td>
<td>&lt; 2.5$</td>
<td>216561</td>
<td>0.999741</td>
</tr>
<tr>
<td>B Layer Hits $\geq 1$</td>
<td>210336</td>
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<td>0.981573</td>
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<tr>
<td>SCT Hits $\geq 6$</td>
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<td>0.994837</td>
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</tr>
<tr>
<td>Pixel Hits $\geq 1$</td>
<td>209250</td>
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<tr>
<td>$</td>
<td>vtx_d_0</td>
<td>&lt; 1.5\text{mm}$</td>
<td>208588</td>
<td>0.996836</td>
</tr>
<tr>
<td>$</td>
<td>vtx_z_0\sin(\theta)</td>
<td>&lt; 1.5\text{mm}$</td>
<td>208394</td>
<td>0.99907</td>
</tr>
</tbody>
</table>

Table 3.1: Absolute number of passing tracks as well as the relative change from the previous cut for both the 900 GeV data and Monte Carlo.
CHAPTER IV

Analysis and Results

The goal is to produce a measurement of the number of charged particles per unit of pseudorapidity per event in ATLAS at 900 GeV, $1/N_{ev} \cdot dN_{ch}/d\eta$. A distribution of track multiplicity per event, $1/N_{ev} \cdot dN_{ev}/dN_{ch}$, was also produced. These were done by counting the number of reconstructed tracks passing the previously mentioned selections. The number of tracks in an event is taken to be equal to the number of charged particles in the event. Thus, the number of tracks in an event $N_{\text{tracks}}$ and the number of charged particles in an event $N_{\text{ch}}$ are used somewhat interchangeably. All uncertainties in histograms are statistical.

4.1 Corrections

The general idea with corrections is to convert a distribution, in say $\eta$, of tracks after reconstruction and selections to a distribution representing the charged particles present in the event. In other words, reverse the effects of reconstruction efficiencies and biases introduced from triggering and selections. In the simplest form, a correction can be computed by inverting the efficiency of the trigger, for instance, as a function of the interesting variable. Corrections are then applied as a weight to events or tracks.

As noted earlier, the trigger efficiency was nearly 100% and flat across the relevant
\( \eta \) for this analysis. Thus no correction was made for the trigger.

Unfortunately, the MC samples used, converted to MTree data files for analysis, did not have truth track information. Thus shape corrections for cuts and track reconstruction efficiency corrections could not be computed and applied to this analysis. However, if this had been available the track reconstruction efficiency would have been calculated as follows:

\[
\epsilon(\eta) = \frac{N_{\text{rec}}(\eta)}{N_{\text{truth}}(\eta)} \tag{4.1}
\]

Where \( N_{\text{rec}}(\eta) \) is the number of reconstructed tracks which match a primary truth charged particle as a function of \( \eta \). \( N_{\text{truth}}(\eta) \) is the number of primary truth charged particles as a function of \( \eta \). This distribution would then be inverted bin by bin to correct the distribution in data. Similarly, corrections for biases from track selections could be estimated using this method.

The following corrections attempt to account for primary vertex reconstruction efficiency. They consider the effect on tracks of the primary vertex requirement on events. Calculating the efficiency is entirely data driven; however, it assumes that a primary vertex is in each triggered event. As long as this assumption is close to being true, the calculated corrections should be worthwhile.

A correction was calculated to account for the effect of the event selection requirement of a primary vertex with three or more associated tracks. \( N_{\text{trig}}(\eta) \) is the number of tracks passing the track selections, except those involving a primary vertex, in a triggered event as a function of the track’s \( \eta \). \( N_{\text{pvtx}}(\eta) \) is the number of tracks passing the track selections, except those involving a primary vertex, in a triggered event which also passes the primary vertex requirement as a function of \( \eta \). A requirement of \( d_0 < 4 \text{mm} \) was put on the tracks in place of the cuts involving the primary vertex. This \( d_0 \) requirement is the same used in the primary vertex reconstruction [2]. The inverse of \( \epsilon_{\text{pvtx}}(\eta) \) is applied as the correction to the \( \eta \)
distribution of tracks. The efficiency was defined as follows:

\[ \epsilon_{pvtx}(\eta) = \frac{N_{trig}(\eta)}{N_{pvtx}(\eta)} \]  

(4.2)

Figure 4.1: The probability that a track with \( \eta \) will be in an event with a reconstructed primary vertex.

The probability a track with measured \( \eta \) is in an event with a reconstructed vertex for both data and Monte Carlo in \( \eta \) can be found in Fig. 4.1. This was computed by dividing bin by bin the distribution of tracks in \( \eta \) in events passing both the trigger and primary vertex requirement by the distribution of tracks in \( \eta \) in events passing the trigger. Assuming every triggered event should have a primary vertex, this distribution should depend on the primary vertex reconstruction efficiency. One can see that in the data there is a slightly lower efficiency than in the Monte Carlo; however, the shape of the distributions are very similar.

In Fig. 4.2 the corrections for data and Monte Carlo were computed separately and then normalized by their respective number of events passing the trigger and primary vertex requirement. The data is generally lower than the MC. This could be due to dead parts of the detector not accounted for in the MC, inefficiencies in vertex reconstruction that could not be picked up using this method of correction, or differences in track reconstruction efficiencies. Again, unfortunately the correction
Figure 4.2: Both Monte Carlo events and data corrected distributions of the number of tracks in $\Delta \eta$ per event as a function of $\eta$.

Figure 4.3: Unweighted distribution of the raw number of tracks as a function of $\eta$. Monte Carlo is scaled to the data for shape comparison.
calculated in this way cannot be compared with Monte Carlo truth information.

Similar to the efficiency in $\eta$, an efficiency in $\phi$ was produced, Fig. 4.4. This is quite flat. The bumps in the data are most likely due to statistical fluctuations. Since the correction is flat it would do nothing to fix the large variations in the track distribution over $\phi$, Fig. 4.5, in what should be a relatively flat distribution. It is expected that a correction for track reconstruction efficiency would make a larger impact on this distribution.

![Primary Vertex Requirement Efficiency](image1)

Figure 4.4: The probability that a track with $\phi$ will be in an event with a reconstructed primary vertex.

![Data vs MC Phi Distribution](image2)

Figure 4.5: Monte Carlo normalized to Data raw number of tracks against $\phi$. Note the disagreement at $\eta \sim -1$ and the drops in track numbers at $\eta \sim -2, -1, 2$. 

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Another correction to take into account the variation in vertex reconstruction as a function of track multiplicity in an event was constructed from the efficiency defined as:

$$\epsilon_{\text{mult}}(N_{\text{mult}}) = \frac{n_{\text{pvtx}}(N_{\text{mult}})}{n_{\text{trig}}(N_{\text{mult}})}$$ (4.3)

Here, $N_{\text{mult}}$ is the number of selected tracks in each event. In this case, the selected tracks must pass all selections except those involving the primary vertex. To make up for this, the tracks were again required to pass an addition cut of $d_0 < 4\text{mm}$. In Eq. 4.3, $n_{\text{pvtx}}$ is the number of events passing the trigger and primary vertex requirement and $n_{\text{trig}}$ is the number of events triggered. In essence, this efficiency is calculated by looking at a single multiplicity bin and computing the fraction of events passing the primary vertex selection and trigger over the number of events passing just the trigger with that multiplicity. This efficiency was calculated out to a multiplicity of 50 and the efficiency for events with larger multiplicity was set to one. In Fig. 4.6 one can see that the efficiency calculated in this way approaches unity after a multiplicity of about 20 selected tracks. Data and Monte Carlo agree well down to a multiplicity of 3, where below that the Monte Carlo has a higher efficiency than seen in Monte Carlo. Events were weighted by the inverse of this efficiency based on the multiplicity of tracks passing the track selection.

A distribution of the track multiplicity in events using the weighted events can be found in Fig. 4.7. For this plot the data and Monte Carlo were both corrected using their respective primary vertex efficiency weighting described above. The distributions were then scaled by the respective number of events passing the trigger and primary vertex selection. The data and Monte Carlo agree out to around a multiplicity of 25, where the data starts to have more events than Monte Carlo at large multiplicities. At the low end of the multiplicity, namely 2, the data is much larger than the Monte Carlo. This could be explained in part by the lower efficiency in the primary vertex reconstruction in data compared to Monte Carlo at low multiplicity.
Figure 4.6: Probability an event with $N$ tracks has a primary vertex, taken to be an estimate of the primary vertex reconstruction efficiency as a function of track multiplicity in events.

However, in the unweighted distribution, Fig. 4.8, the data was also larger than the Monte Carlo at low multiplicity.

The distribution in Fig. 4.7 is the probability distribution for a given event to have a multiplicity $N$ at 900 GeV. Similarly, the distribution in Fig. 4.2 is the probability an event will have a track in a $\Delta \eta$.

Figure 4.7: Normalized event multiplicity weighted by the primary vertex efficiency as a function of track multiplicity.
4.2 Results

The number of tracks per unit $\eta$ per event as a function of $\eta$ was calculated by dividing each bin in Fig. 4.2 by the bin width. This distribution can be found in Fig. 4.9. An average value was computed by taking the mean of each bin to find $1/N_{ev} \cdot dN_{ch}/d\eta = 1.146 \pm 0.141(stat.)$ for $|\eta| < 2.5$. The uncertainty here is purely statistical, based on the variation in the histogram values across the bins. This result
is consistent with the measurement published by the ATLAS group of $1.333 \pm 0.043$ [2]. For the Monte Carlo $1/Ev \cdot dN_{ch}/d\eta = 1.195 \pm 0.121(\text{stat.})$.

In the central region $|\eta| < 1$, the number of tracks is slightly higher. Looking at only this region gives a value for $1/Ev \cdot dN_{ch}/d\eta$ of $1.274 \pm 0.037$ for the data and $1.310 \pm 0.011$ for the Monte Carlo.

The obtained result is expected to be lower than the true number because additional corrections, such as track reconstruction efficiency, which were not performed would increase the effective number of tracks in the data.
APPENDIX A

Additional Plots

Figure A.1: Monte Carlo normalized to Data raw number of tracks against $P_T$. 
Figure A.2: Left: Monte Carlo normalized to Data raw number of tracks against $D_0$. Right: Monte Carlo normalized to Data raw number of tracks against $Z_0 \sin(\theta)$. The $D_0$ plot is before the $Z_0 \sin(\theta)$ requirement.
REFERENCES


